SS – 350

V Semester B.A./B.Sc. Examination, Nov./Dec. 2018
(Semester Scheme) (Repeaters – Prior to 2016-17)
(NS – 2013-14 and Onwards)
Mathematics
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 100

Instruction : *Answer* ***all*** *questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Solve $(y + z)\,dx + (z + x)\,dy + (x + y)\,dz = 0$.

2) Verify the condition for integrability $3x^2\,dx + 3y^2\,dy - (x^3 + y^3 + e^{2z})\,dz = 0$.

3) Form the partial differential equation by eliminating the arbitrary function from $Z = f(x^2 + y^2)$.

4) Solve $\sqrt{p} + \sqrt{q} = 1$.

5) Solve $z - px - qy = 2\sqrt{pq}$.

6) Solve $[D^2 - 4DD' + 4(D')^2]\,Z = 0$.

7) Obtain the expression for $P_2(x)$ using Rodrigues' formula.

8) Express $1 + 2x - 3x^2$ in a series of Legendre polynomials.

9) Show that $J_n(-x) = (-1)^n J_n(x)$.

10) Prove that $J_0'(x) = -J_1(x)$.

11) Write the Bessel's differential equation.

12) Evaluate $\Delta(\log x)$.

13) Prove that $E\,\Delta = \Delta\,E$.

14) Express the polynomial $x^2 + x + 1$ as a factorial polynomial (taking $h = 1$).

15) Write the Newton – Gregory backward interpolation formula.

16) Write the formula for Simpson's $3/8^{th}$ rule.

17) Define mathematical modelling and give an example.

18) How long does it take for a given amount of money to double at 10% per annum compounded annually ?

19) In the case of modelling of projectile motion without air resistance write the expression for time of flight.

20) What are the assumptions to be made in getting a partial differential equation model for a vibrating string ?

P.T.O.

II. Answer **any four** questions : **(4×5=20)**

1) Verify the condition for integrability and solve $(2x^2 + 2xy + 2xz^2 + 1)\,dx + dy + 2zdz = 0$.

2) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

3) Form a partial differential equation by eliminating the function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

4) Solve by Charpit's method $px + qy = pq$.

5) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x$.

6) Reduce $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to a canonical form.

OR

Solve $p + q = \sin x + \sin y$.

III. Answer **any three** questions : **(3×5=15)**

1) Prove that $\frac{1 - t^2}{(1 - 2xt + t^2)^{3/2}} = \sum_{n=0}^{\infty} (2n + 1) P_n(x)\, t^n$.

2) Prove that $\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0$ if $m \neq n$.

3) Prove that $P_n(-x) = (-1)^n P_n(x)$.

4) Prove the following :

a) $\cos(x\sin\theta) = J_0(x) + 2 \sum_{1}^{\infty} J_{2n}(x) \cos 2n\theta$.

b) $\sin(x\sin\theta) = 2 \sum_{1}^{\infty} J_{2n-1}(x) \sin[(2n - 1)\theta]$.

5) Prove that $J_{n+1}(x) + J_{n-1}(x) = \frac{2n}{x} J_n(x)$.

IV. Answer **any four** questions : **(4×5=20)**

1) Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$, $y_7 = 17$, calculate $\Delta^4 y_3$.

2) Find a polynomial which takes the values :

x	3	4	5	6	7
y	6	24	60	120	210

using Newton-Gregory forward interpolation formula.

3) By the method of separation of symbols prove that

$$u_0 - u_1 + u_2 - u_3 + \ldots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \ldots.$$

4) Find the value of f(10) from the data using Newton's divided difference table :

x	5	6	9	11
f(x)	12	13	14	16

5) Find $\frac{dy}{dx}$ at x = 54 from the following table :

x	50	51	52	53	54
y	3.6840	3.7084	3.7325	3.7563	3.7798

6) Evaluate $\int_1^2 \frac{dx}{x}$ using Simpson's 1/3rd rule taking four subintervals.

V. Answer **any three** questions : **(3×5=15)**

1) In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will it cross 2,00,000 if the rate of growth of bacteria is proportional to the number present ?

2) Uranium disintegrates at a rate proportional to the amount present at any instant. If m_1 and m_2 grams of uranium are present at time t_1 and t_2. Show that half life of uranium is $\frac{(t_1 - t_2)\log 2}{\log\left(m_1/m_2\right)}$.

3) A generator having e.m.f. 100 V is connected in series with 20 Ω resistor and inductor of 4 H. Determine the current if i(0) = 0. Find i for t = 0.2 sec.

4) From the differential equation of the free damped motion in the case of mass-spring-dashpot and discuss :
   i) Over damped and
   ii) Critically damped cases.

5) A projectile when thrown at an angle $\tan^{-1}$ (3/4) falls 40 metres short of the target. When it is fired at an angle of 45°, it falls 50 m beyond the target. Find the distance of the target from the point of projection.